Mathematics Teaching and Learning: Where to?

By Doug Clarke

Introduction

It is an exciting time to be involved in mathematics education in Australia. Governments at national and state levels are giving particular focus to the teaching and learning of mathematics at present, resulting in a number of promising initiatives at school and system levels. In this article, I provide a brief background on the policy context for mathematics education, identify a number of issues which demand attention, and give a brief snapshot of promising developments in a project funded by the Catholic Education Office Melbourne (CEOM), which is seeking to support teachers as they address many of these issues.

There is occasionally vigorous debate regarding the differences between mathematics (the discipline), mathematics (the school subject) and numeracy. There appears to be fairly widespread acceptance in Australia among professional associations, systems and schools that to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life. There is probably less attention given to the statement by the Australian Association of Mathematics Teachers that numeracy is a fundamental component of learning, discourse and critique across *all areas of the curriculum*. While acknowledging the differences, mathematics and numeracy will be used somewhat interchangeably in this article.

Background

In 2008, the Australian Government outlined a National Action Plan on Literacy and Numeracy, highlighting the importance of literacy and numeracy as the foundations upon which further learning is built. They noted that the key influences on children's literacy and numeracy learning outcomes centre on effective teaching practices, leadership, whole-school approaches, and the use of data to drive improvement.

Subsequently, the Government invited systems to propose pilot programs in disadvantaged school communities to improve literacy and numeracy outcomes for students. This focus on disadvantaged communities was understandable, given the widespread concern that students from these communities are far less likely to meet desirable goals in these important content areas. The focus of government initiatives on effective teaching practice (so closely linked, of course, to teacher professional knowledge), the importance of leadership and whole-school approaches to change, and the collection and use of appropriate evidence, all find support in the literature.

In May 2008, the *National Numeracy Review Report* was released by the Council of Australian Governments. It was a privilege to be a member of the panel which wrote the report. I will draw on some of the recommendations, as relevant in the discussion which follows.

Some challenges we face in the teaching of mathematics

A national shortage of mathematics teachers. In an increasingly technological society, the need for students to continue their study of mathematics to senior secondary and on to university becomes even greater. It is a cause of great concern that the participation of students in tertiary mathematics continues to decline. The availability of qualified mathematics teachers is likely to be a contributing factor in this decline.

Australia shares this problem with many other countries. The shortage has meant that increasingly secondary mathematics is being taught 'out of field' (e.g. teachers trained in physical education being given a Year 8 maths class). The shortage also results in the teachers who have strong qualifications in mathematics being assigned to senior secondary classes only, with an obvious impact on the lower year levels. There is also a temptation at government level to address this shortage by recruiting

Published by the Catholic Education Office Melbourne in *Learning Matters*, volume 14, number 1, 2009, pp. 3–8

individuals who have used mathematics in industry or in trades, and offering them short teacher preparation courses. The teaching of mathematics involves much more than content knowledge, and short-term benefits of such recruitment are likely to have lasting negative effects in the medium and long term.

A nation of adults with negative views of school mathematics. As part of the Contemporary Teaching and Learning of Mathematics project (CTLM; funded by CEOM), the ACU team and the CEOM have been conducting parent information evenings in all project schools. During these evenings, we invite parents to reflect on personal experience of school mathematics. I would estimate that around 90 per cent of these adults view their experiences negatively. This reflects my experiences in such settings over the past 25 years, and it is of great concern if these attitudes are being passed on to children. Well-meaning parents are also spending funds on expensive computer packages or tutoring programs, believing that they are doing so in their children's interests. These programs tend to focus on rote learning, and teachers have frequently mentioned to me that students who participate in such activities often lack the capacity to apply their knowledge in practical contexts. During the CTLM parent evenings, we have been emphasising that the interests of children would be better served by parents taking time to involve their children in enjoyable everyday activities with a mathematical component – while cooking, shopping, travelling, and playing board, dice and card games. School is a long day for students, and more of the same, at least at primary level, is likely to be counter-productive.

A middle school curriculum of routine tasks. The TIMSS video study of Year 8 classrooms in Australia (Hollingsworth et al. 2003) noted there was an over-emphasis on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on were rare. They noted a syndrome of shallow teaching, where students were asked to follow procedures without reasons. These findings tie in with the experiences of parents as mentioned above. The *National Numeracy Review Report* (COAG 2008) recommended:

> that from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking (p. 31).

A crowded curriculum. A survey of primary school principals, reported in *The Age* in 2007, indicated that numeracy was timetabled for approximately 18 per cent of the day, with literacy 38 per cent. The considerable discrepancy between these two is food for thought, as is my anecdotal experience that the time actually devoted to mathematics/numeracy can be considerably less than this, with the many pressures of important extra-curricular activities. I also despair somewhat when schools express a desire for projects to 'leave them alone' in June because they are focusing on reporting. This is a worrying perspective.

The crowded curriculum issue is also tied in to the previous point, as the pressure to 'cover' (rather than 'uncover') the curriculum leads to hasty treatment of important ideas, with little residual benefit. It is encouraging to hear that the early work on a national curriculum is seeking to make some hard decisions in reducing both the size of curriculum guidelines and the number of topics to be addressed. As part of this process, it will be important to consider the ways in which the 'basics' are changing. What formed the basics in the 1950s, 1960s or 1970s must be very different from what these are today.

Lack of appropriate balance in forms of assessment. The National Assessment Program – Literacy and Numeracy (NAPLAN) can provide important data to teachers, schools, and systems. Used appropriately, the data can provide information on patterns of strength and areas requiring attention (e.g. tasks on which our school performs particularly better or poorer than other schools). However, the weaknesses of multiple-choice items administered in May with results not available until at least September are obvious. The *National Numeracy Review Report* recommended that:

> a balanced view be taken of the relative contributions to effective student learning of systemic assessment programs and high quality classroom assessment in the allocation of resources to develop and support each (p. 42).

It has long been recognised in *literacy* that one-to-one assessment opportunities are crucial, and the development and use of such interviews in mathematics (e.g. in the CEOM Success in Numeracy Education program and the Early Numeracy Interview) have been a key factor in teacher professional learning and assessment in the past 10 years. Such interview protocols tied to research-based *growth points* have meant that teachers have a far better sense of what students know and can do, as well as a sense of a developmental continuum or learning trajectory in key curriculum areas. The Cockroft Report in the UK many years ago noted the remark that 'no one has ever grown taller by being measured' (1982, para. 421) and of course this emphasises the importance of assessment being the prelude to action on the part of schools and teachers.

The need to consider carefully ability grouping in maths. Clarke and Clarke (2008) suggested nine reasons why 'the time is up' for ability grouping in maths:

1. The research evidence is clear that generally any benefits which accrue from ability grouping are only to very high achievers, with a negative impact on average and low attaining students.
2. International testing data show that ability grouping has an overall negative effect on a country's performance.
3. Ability grouping can lead to the mistaken perception that individual differences are no longer an issue.
4. Many schools assign their least-qualified teachers to the lower ability classes.
5. Teachers of lower classes often have low expectations of what students can do.
6. Students are often grouped according to narrow criteria, and it is assumed that these classes are appropriate for all kinds of tasks and all content areas.
7. Despite claims of flexibility, lower ability classes are very hard to leave.
8. There is a range of strategies which can help teachers to cater for mixed abilities.
9. Ability grouping cannot be supported in the interests of social justice.

This view found support in the *National Numeracy Review Report*, which recommended that

> the use of ability grouping in primary and junior secondary schooling be discouraged given the evidence that it contributes to negative learning and attitudinal outcomes for less well achieving students and yields little positive benefit for others, thus risking our human capital goals (p. 49).

Within-class ability grouping for mathematics is another issue for consideration. A large meta-analysis of many studies by Lou and his colleagues in the United States found that there were no significant differences between ability grouping and mixed-ability grouping in mathematics, compared with significant differences in reading in favour of homogeneous groups.

There has been a tendency in Australian schools to take on practices in mathematics which are believed to be effective in literacy (such as within-class ability groups), assuming that what works in one curriculum area must work in another. This view demands closer examination.

From my experience in working with teachers and my reading of the research, I lean towards flexible approaches where the students work on open tasks in mixed-ability mathematics for much of the time, with the teacher occasionally pulling a group aside for extra support or extension work.

Looking to a positive future in the teaching of mathematics

The challenges we face are clear and substantial. However, we can be encouraged by a number of current developments in mathematics education:

A willingness to look and learn from other countries. International comparisons need to be viewed carefully. It has been claimed that one can always find a high-performing country with practices to one's liking. Given Finland's relative success in international studies of mathematics achievement (e.g. in the Programme for International Student Assessment (PISA)), other countries have been looking to that country to see what lessons can be learned. The features which stand out to me from my reading are the following:

- All teachers have at least a masters' degree.
- Preparation for teaching has a research focus.
- Teaching has high status (more selective entry than law or medicine).
- There is a strong commitment to high quality teacher professional learning.
- There is considerable teacher autonomy.
- Standardised testing is shunned.
- Schools have a pleasant environment ('easygoing').
- Little homework is set.
- There is a strong emphasis on supporting those who need additional assistance.

A recognition that teacher knowledge is the key. We all acknowledge the crucial role the teacher plays in effective learning. But professional learning programs in mathematics teaching are increasingly focusing on *pedagogical content knowledge* (PCK), reflecting the increasing evidence that this is what makes the difference in student learning. This term originated with Shulman in the 1970s. He described PCK as an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of the most frequently taught topics and lessons … the most powerful analogies, illustrations, examples, explanations, and demonstrations related to the topic one teaches … the ways of representing and formulating the subject that make it comprehensible for others. A consideration of these aspects of PCK makes it clear why short teacher training programs for, say, engineers, are hardly likely to provide the kinds of knowledge needed to teach mathematics effectively.

Enhanced teacher pedagogical content knowledge is a stated aim of the CEOM's CTLM project. As part of CTLM, we developed questionnaire items where teachers were asked to comment on particular mathematics classroom scenarios, for example:

- suggesting what the nature of student difficulty might be, based on a work sample
- suggesting appropriate activities to build student understanding of a particular maths concept
- identifying the 'big ideas' in a particular piece of content.

By the end of 2008 we were delighted to see a considerable improvement in the PCK of the CTLM teachers, as measured by these items, as a result of teachers' involvement in professional learning, within-class support, and their dedication to improving their practice. We admit that teachers found these items quite threatening initially, but most were delighted by the greater ease with which they responded to the items towards the end of the first year of the project.

A valuing of student methods. Many adults recollect that in their school mathematics there was always 'one right answer' (and method) to a given problem, as with the TIMSS video study above (Hollingworth et al. 2003). Increasingly, professional learning programs are emphasising that students can be encouraged to develop their own methods of solving problems, with a belief that if students understand their own method (or that of another student), and the method is mathematically valid, efficient and can be generalised, it can be very empowering for students, and their understanding is likely to be more substantial. Sharing of such methods can lead to enthusiastic discussion in classrooms, and appropriate transfer of methods by students to new problems.

Related to this development is the increasing evidence that delaying the teaching of standard written algorithms[1] is desirable in the junior and middle years of primary school. Data from a study by Northcote and McIntosh (1999) showed that approximately 86 per cent of all calculations which adults do are mental. Teachers are now devoting greater time to valuing and building upon a variety of student mental strategies, and giving less emphasis to the less commonly-used conventional written methods, which can interfere with the development of efficient mental strategies. The 'grandfather' of mathematical problem-solving, George Polya, said many years ago that it was better to solve one problem five different ways than five problems. This approach is much more evident in Victorian primary schools than it once was.

Classrooms (particularly in primary school) where there is lot of talking about, enjoying and doing substantial mathematics. It is a considerable privilege to work with dedicated and enthusiastic teachers in the CTLM project. In finishing this article, I thought that I might tell one story of the kinds of activities in which these teachers and their students are engaged, and which exemplify many of the points made in this article.

As most readers would be aware, a pedometer is something which can worn on a belt, and which indicates the number of steps an individual takes while walking or running. Pedometers have the potential for very worthwhile mathematical investigations and problem-solving. In working in schools, teachers and I have given pedometers to three students, explaining that we will be interested in how many steps the three students take by the time we meet later in the day. I also wear a pedometer, but invariably the students have taken many more steps than I by the time we meet!

During class, we work through a range of activities of the following kinds:

- We estimate and then check how many steps the students might take as they walk the length of the corridor, do two laps of the classroom etc.
- We look at the pedometer readings (e.g. 4892, 3918, and 5711) and the students are asked to calculate what the pedometers would show if the wearer did 10 more steps, 100 more steps, 100 fewer steps, etc.
- We explain that it is recommended that adults take 10,000 steps per day if they wish to remain fit. I then challenge the students to calculate (mentally if they can) how many more steps each of the pedometer wearers will need to take in order to get to 10,000. Increasingly, students are likely to use efficient and effective 'building up' strategies. Andrew (pictured), a Year 2 student at St Anthony's, Glen Huntly, worked out how many more steps would be needed to go from 2124 to 10,000 *in his head*. He explained his method, and I drew a picture on the whiteboard so other students could understand it. As the diagram shows, he built up, by adding 6 to the ones, 70 to the tens, and so on. Of course, this was most impressive thinking from a Year 2 student. The empty number line I drew is quite often a helpful pictorial representation of students' mental strategies in these kinds of tasks.

The pedometer activities have a number of features of mathematics teaching and learning which are increasingly being identified in professional learning settings:

- the use of a motivational and enjoyable context (here, the pedometer) to engage students in mathematics
- a focus on estimation (over 60 per cent of all calculations which adults do require only an estimate)
- the valuing of a variety of student mental calculation methods with a focus on visual or other written ways of representing them.

As this article makes clear, mathematics teaching in Australia continues to face major challenges. However, the resolve of governments and systems to provide substantial funding for teacher

[1] Algorithms are step-by-step procedures such as the decomposition method of subtraction, where, for example, students working out 475 – 289 trade one of the 7 tens for 10 ones (crossing out the 7 and replacing it with 6), and thereby changing 5 ones into 15 ones, from which they easily subtract 9 ones.

professional learning programs, with a particular emphasis on those students in greatest need, provides hope of a far more positive future.

References

Council of Australian Governments (COAG) 2008, *National Numeracy Review Report*, (Commissioned by the Human Capital Working Group, Council of Australian Governments), COAG, Canberra.

Clarke, DM 2005, 'Written algorithms in the primary years: Undoing the good work?', in M Coupland, J Anderson & T Spencer (Eds), *Making Mathematics Vital*, Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers held in Sydney in January 2005, AAMT, Adelaide, 93–98.

Clarke, DM & Clarke, BA 2008, 'Is time up for ability grouping?', *EQ*, Autumn, 31–33.

Cockroft, WH (Chair) 1982, *Mathematics Counts*, Report of the Committee of Inquiry into the Teaching of Mathematics in Schools, Her Majesty's Stationery Office, London, <www.dg.dial.pipex.com/documents/docs1/cockcroft.shtml>.

Hollingsworth, H, Lokan J & McCrae, B 2003, *Teaching Mathematics in Australia: Results from the TIMSS 1999 Video Study*, Australian Council for Educational Research, Camberwell, Victoria.

Lou, Y, Abrami, PC, Spence, JC, Poulsen, C, Chambers, B & d'Apollonia, S 1996, 'Within-class grouping: A meta-analysis', *Review of Educational Research*, 66 (4), 423–458.

Northcote, M & McIntosh, M 1999, 'What mathematics do adults really do in everyday life?', *Australian Primary Mathematics Classroom*, 4 (1), 19–21.

Shulman, LS 1986, 'Those who understand: Knowledge growth in teaching', *Educational Researcher*, 15, 4–14.

Doug Clarke is a Professor of Mathematics Education at the Australian Catholic University (Victoria), where he directs the Mathematics Teaching and Learning Centre. Recently, Doug was a member of the Expert Panel which prepared the Report of the National Numeracy Review for the Council of Australian Governments.